

Unit 6

Generating Sequences 8as1

Linear Sequence: Terms in the sequence increase by the same amount each time

Example A: 5 7 9 11 13...

+2 +2

Term-to-term rule: A rule to find a term of a sequence, given the previous term

Example: The term-to-term rule for Example A is +2

Position-to-term rule: The rule that allows any term in a sequence to be calculated, given its position number

Example: The position-to term sequence is: $2 \times \text{position number} + 3$ *Plug in!*

Position Number	1	2	3	4	5
Term	5	$2 \times 2 + 3$	9	$2 \times 4 + 3$	13

7 11

Example #1: The first term of a sequence is 4. The **term-to-term rule** of the sequence is 'subtract 3'. Write the first three terms of the sequence.

First term: 4, 1, -2
-3 -3

Example #2: The **position-to-term rule** of a sequence is: $\text{term} = 4 \times \text{position number} + 1$. Work out the first three terms of the sequence.

Position Number	1	2	3
Term	$4 \times 1 + 1$	$4 \times 2 + 1$	$4 \times 3 + 1$

Practice! Find the first three terms

- a. First Term: 1 term-to-term rule: 'add 5'
- b. First Term: 20 term-to-term rule: 'subtract 4'
- c. $\text{Term} = 6 \times \text{position number}$
- d. $\text{Term} = \text{position number} - 4$
- e. First term: -3 term-to-term rule: 'subtract 6'

Practice!

Finding Rules for sequences 8as1

Example A:

Pattern 1



5 Dots

Pattern 2



7 Dots

Pattern 3



9 Dots

Term-to-term Rule:

+2

Position-to-term Rule:

$$\frac{2 \times n + 3}{2(1) + 3} = 5$$

$$\frac{2(1) + 3}{2 + 3} = 5$$

Try: $2n + 3$

$$2(2) + 3 = 7 \checkmark$$

Find the **Term-to-term Rule** and work out the **Position-to-term Rule**

Example #1: 3, 6, 9, 12, ...

A. Term-to-term Rule:

+3

B. Position-to-term Rule

Step One: Create a table with the position number, term, and multiples

Position Number	1	2	3	4
Term	3	6	9	12
Multiples of	3	3	3	3

Step Two: Fill in the rule **term = 3 x position number**

Step Three: Check if the rule works with the first terms

$$3 \times (1) = 3$$

$$3 \times (2) = 6$$

$$3 \times (3) = 9$$

$$3 \times (4) = 12 \checkmark$$

Example #2: 5, 9, 13, 17, ...

A. Term-to-term Rule: $+4$

B. Position-to-term Rule

Step One: Create a table with the position number, term, and multiples

Position Number	1	2	3	4
Term	5	9	13	17
Multiples of	$\times 4$	$\times 4$	$\times 4$	$\times 4$

Step Two: Fill in the rule term = 4 x position number $?$

Step Three: Check if the rule with the first terms

Example #3: 6, 7, 8, 9

A. Term-to-term Rule:

$$4 \times (1) + \underline{\quad} = 5 \quad , \quad 4pn + 1$$

$$\underbrace{4}_{4} \times \underbrace{1}_{1} = 5$$

B. Position-to-term Rule

Step One: Create a table with the position number, term, and multiples

Position Number	1	2	3	4
Term	6	7	8	9
Multiples of	1	2	3	4

Step Two: Fill in the rule term = 1 x position number $?$

Step Three: Check if the rule with the first terms

Example #4:

$$1 \times (1) + \underline{\quad} = 6 \quad , \quad 1pn + 4$$

$$\underbrace{1}_{1} + \underbrace{4}_{4} = 6$$

1, 4, 7, 10, ...

A. Term-to-term Rule:

B. Position-to-term Rule

Step One: Create a table with the position number, term, and multiples

Position Number	1	2	3	4
Term	1	4	7	10
Multiples of	3	3	3	3

Step Two: Fill in the rule term = 3 x position number $?$

Step Three: Check if the rule with the first terms

Class/Home Work: #2-5

$$1 \times (3) - \underline{\quad} = 1 \quad , \quad 3pn - 2$$

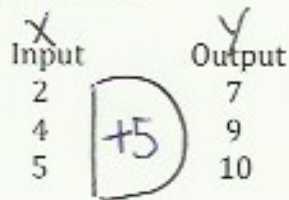
$$\underbrace{3}_{3} - \underline{2} = +1$$

Using Functions and Mapping 8as3

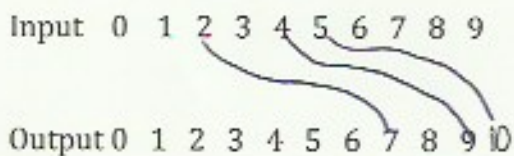
Function: Relationship between two sets of numbers

Three ways to represent the function

1) Function Machine:



2) Mapping Diagram: Input number map the output numbers



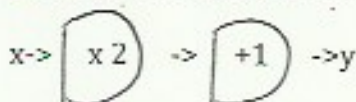
3) Algebraic Equation:

- Letter "x" represents input number
- Letter "y" represents output number

$$x + 5 = y$$

Example #1:

A. Copy and complete the table of values for this function machine



x	1	2	3	4
y	2	5	7	9

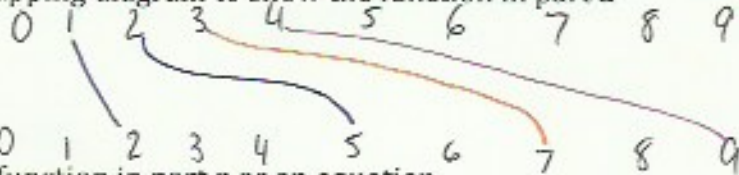
1×2
 $2 + 1 = 2$

2×2
 $4 + 1 = 5$

2×3
 $6 + 1 = 7$

4×2
 $8 + 1 = 9$

B. Draw a mapping diagram to show the function in part a



C. Write the function in part a as an equation

$$x \cdot 2 + 1 = y$$

$$\downarrow$$

$$2x + 1 = y$$

Constructing Linear Expressions 8ae3

Algebraic Expressions: Using letters to represent an unknown number

$$N+5$$

Variable: Letter used in an algebraic expression

Linear Expression: An expression with at least one variable

*** n^2 is **NOT** a linear expression; it is only multiplied by itself

Example #1:

A) Write the number that is five more than the mystery number $\rightarrow 5+n$

B) Write the number that is three times the mystery number $\rightarrow 3 \times n \rightarrow 3n$

C) Write the mystery number multiplied by itself $\rightarrow n \cdot n \rightarrow n^2$

Example #2:

Victoria thinks of a number, x . Write down an expression Victoria gets when:

A) She subtracts five from the number $\rightarrow n-5$

B) She doubles the number and adds three $\rightarrow n \times 2 + 3 \rightarrow 2n+3$

C) Divides the number by three and adds two $\rightarrow \frac{n}{3} + 2$

D) Adds 2 to the number and then multiplies by 4 $\rightarrow (2+n) \times 4 \rightarrow 4(2+n)$

Class Work 1-4

White Board Practice

Deriving and using formulas 8nc10

Formula: mathematical rule that shows the relationship between two or more quantities (variables)

Derive: write a formula

Using Formula

***Use order of operations: Please Excuse my Dear Aunt Sally

Brackets (Parenthesis)
Indices (Exponents)
Division
Multiplication } Left to Right
Addition
Subtraction } Left to Right

Practice #1:

$$\begin{array}{l} 2 - 1 \times (5 + 3) = \\ \quad \downarrow \quad \downarrow \quad \downarrow \\ 2 - 1 \times 8 = \\ \quad \downarrow \quad \downarrow \\ 2 - 8 = \underline{-6} \end{array}$$

Practice #2:

$$\begin{array}{l} 2 - 1 \times 5 + 3 = \\ \downarrow \quad \downarrow \quad \downarrow \\ 2 - 5 + 3 = \\ \quad \downarrow \quad \downarrow \\ -3 + 3 = \underline{0} \end{array}$$

Practice #3:

$$\begin{array}{l} 44 - 6^2 \div 4 \times 2 = \\ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ 44 - 36 \div 4 \times 2 = \\ \quad \downarrow \quad \downarrow \quad \downarrow \\ 44 - 9 \times 2 = \\ \quad \downarrow \quad \downarrow \\ 44 - 18 = \underline{26} \end{array}$$

Practice!

$$6 \div 2 (1 + 2) = ?$$

9 or 1

WHAT IS THE ANSWER?

$$7 + 7 \div 7 + 7 \times 7 - 7$$

Like term worksheet!

Collecting Like Terms 8ae2

A Short Story:

Once upon a time there were these groups of families scattered across the country in a kingdom far, far, away. The different family groups all decided to find the other members of their family so that each group could hold a giant family reunion. Each family could identify their other family members because they all had the same last name. There was even a family group with no last name! Now let's use our math properties to help these families join up with each other for their big family reunions!

$$\underline{2x^2} + \underline{3xy} + \underline{3x} + \textcircled{6} + \underline{5x^2} + \textcircled{3} + \downarrow 4y^2 + \underline{2x} + \underline{xy} + \textcircled{1}$$

$2x^2 + 5x^2$
 $\quad \quad \quad \vee$
 $\quad \quad \quad 7x^2$

$3xy + xy$
 $3xy + 1xy$
 $\quad \quad \quad \vee$
 $\quad \quad \quad 4xy$

$3x + 2x$
 $\quad \quad \quad \vee$
 $\quad \quad \quad 5x$

$6 + 3 + 1$
 $\quad \quad \quad \vee \quad \downarrow$
 $\quad \quad \quad 9 + 1$
 $\quad \quad \quad \vee$
 $\quad \quad \quad 10$

$4y^2$

[→ rewrite

$$7x^2 + 4y^2 + 4xy + 5x + 10$$

Brackets ()
 indices/exponents
 division \rightarrow Left to Right
 multiplication \rightarrow Left to Right
 addition \rightarrow Left to Right
 subtraction \rightarrow Left to Right

Numeric values and a variable

$$\begin{array}{l} 3 + 5 \times n \\ \downarrow \quad \downarrow \\ 3 + 5n \end{array} \Rightarrow \text{rewrite } 5n + 3 \text{ (variable before integer)}$$

$$\begin{array}{l} 5 \div n + 3 - 2 \\ \downarrow \quad \downarrow \\ \frac{5}{n} + 3 \end{array}$$

$$\begin{array}{l} 3n - 4 + 2 \\ \downarrow \quad \downarrow \\ 3n - 2 \end{array}$$

Numeric values and several variables

$$\begin{array}{l} d + 5 + 2 \times c \\ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ d + 5 + 2c \end{array} \Rightarrow \text{rewrite } 2c + d + 5 \text{ (alphabetical order)}$$

$$\begin{array}{l} d + 3d + 4 \\ \downarrow \quad \downarrow \\ 4d + 4 \end{array}$$

$$\begin{array}{l} 4 + 2x^2 + 3x + 1x \\ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ 4 + 2x^2 + 4x \end{array} \Rightarrow \text{rewrite } 2x^2 + 4x + 4 \text{ (exponent first)}$$

First set of cards pg 97

Simplifying like terms

Simplify: transform an expression by writing it in a different way

Example: $2n+3$ same as $2xn+3$ same as $3+2n$ same as $3+2 \times n$

Which one is the same to $5n-3$? $3-5n$ or $-3+5n$
 $-5n+3$ $+5n-3$

Expanding Brackets

Expand: multiply each term inside the brackets by the term outside the bracket
(Distributive example with Integers:

4(8+3) Using BIDMAS

$$4(11) = 44$$

4(8+3) Expanding Brackets

$$\begin{array}{r} 4 \cdot 8 \quad 4 \cdot 3 \\ \hline 32 + 12 = 44 \end{array}$$

$$\begin{array}{r} d^2(d-1) \\ \hline d^2 \cdot d \quad d^2 \cdot (-1) \\ \hline d^3 - d^2 \end{array}$$

Example With Variables:

$$3(b+5)$$

$$\begin{array}{r} 3 \cdot b \quad 3 \cdot 5 \\ \hline 3b + 15 \end{array}$$

$$a(a-3)$$

$$\begin{array}{r} a \cdot a \quad a \cdot (-3) \\ \hline a^2 - 3a \end{array}$$

$$5(2c-1)$$

$$\begin{array}{r} 5 \cdot 2c \quad 5 \cdot (-1) \\ \hline 10c - 5 \end{array}$$

$$6(3-4e)$$

$$\begin{array}{r} 6 \cdot 3 \quad 6 \cdot (-4e) \\ \hline 18 - 24e \end{array}$$

$$6(6-d)$$

$$\begin{array}{r} 6 \cdot 6 \quad 6 \cdot (-d) \\ \hline 36 - 6d \end{array}$$

Example with several expansion

$$4(2x+3x^2) - x(6+x)$$

$$\begin{array}{r} 4 \cdot 2x \quad 4 \cdot 3x^2 \quad -x \cdot 6 \quad -x \cdot x \\ \hline 8x + 12x^2 \quad -6x \quad -x^2 \\ \hline 8x - 6x \quad 12x^2 - x^2 \\ \hline 2x + 11x^2 \end{array}$$

$$2(x+3) + 3(x+4)$$

$$\begin{array}{r} 2 \cdot x \quad 2 \cdot 3 \quad 3 \cdot x \quad 3 \cdot 4 \\ \hline 2x + 6 + 3x + 12 \\ \hline 2x + 3x + 6 + 12 \\ \hline 5x + 18 \end{array}$$

Practice:

a) $8(x+5) - 3(2x+7)$

b) $a(2b+c) + b(3c-2a)$

c) $2y(y+5x) + x(3x+4y)$

Solving and Constructing Equations

With equations that equal 0

$$\star 3(b+5)=0$$

$$5(2c-1)=0$$

Practice
 $6(3-4e)=0$

$$6(6-d)=0$$

$$3(b+5) = 0$$

$$\begin{array}{r} \curvearrowright \\ 3 \cdot b \quad 3 \cdot 5 \\ \downarrow \quad \downarrow \\ 3b + 15 - 15 = 0 - 15 \\ \downarrow \quad \downarrow \\ 3b \quad 0 = -15 \\ \downarrow \\ \frac{3b}{3} = \frac{-15}{3} \\ b = -5 \end{array}$$

$$\star 4(2x+2)-1(6+x)$$

Practice
 $2(x+3)+3(x+4)$

$$4(2x+2)-1(6+x) = 0$$

$$\begin{array}{r} \curvearrowright \quad \curvearrowright \\ 4 \cdot 2x \quad 4 \cdot 2 \quad -x \cdot 6 \quad -1 \cdot x \\ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ \underline{8x} + \underline{8} \quad \underline{-6x} \quad \underline{-x} \\ 8x - 6x - x + 8 - 8 \\ \downarrow \quad \downarrow \\ 2x - x + 0 \\ \downarrow \\ x + 0 \\ x = -8 \end{array}$$

With equations that equal something other than 0

$$\cancel{8} \quad 6(x+1)=72 \quad 6x-3=9 \quad 3(x+5)=27$$

$$\begin{array}{r|l} 6(x+1) & = 72 \\ \hline 6 \cdot x & \quad \quad \quad \downarrow \\ 6 \cdot 1 & \quad \quad \quad \downarrow \\ \hline 6x + 6 & = 72 - 6 \\ -6 & \quad \quad \quad \downarrow \\ \hline 6x + 0 & = 66 \\ \cancel{6x} & \quad \quad \quad \downarrow \\ \hline & = \frac{66}{6} \\ & \quad \quad \quad \downarrow \\ & = 11 \end{array}$$

With Equation that equal a variable

$$\cancel{*} \quad x+35=5x-13 \quad 4(y-3)=2y+2 \quad 8y-5=3(y+5) \quad 2(y+6)=4(y-3)$$

$$\begin{array}{r|l} x+35 & = 5x-13 \\ \hline x+35 + 13 & = 5x-13 + 13 \\ \downarrow & \quad \quad \quad \downarrow \\ x+48 & = 5x+0 \\ \downarrow & \quad \quad \quad \downarrow \\ 48 & = 4x \\ \hline 12 & = x \end{array} \quad \begin{array}{l} \text{Check!} \\ (12)+35 = 5(12)-13 \\ 47 = 60-13 \\ \downarrow \\ 47 = 47 \checkmark \end{array}$$

Arithmetic with Integers

Integers:

- Whole numbers
- Negative or positive
- Includes zero
- Can be displayed in the number line

Adding and subtracting positive and negative integers

How it works:

Same signs add and keep

$$\textcircled{A} \quad \begin{array}{r} 4+3 \\ -4-3 = -7 \end{array} \quad \begin{array}{r} 5+2 \\ +5+2 = +7 \end{array} \quad \textcircled{C}$$

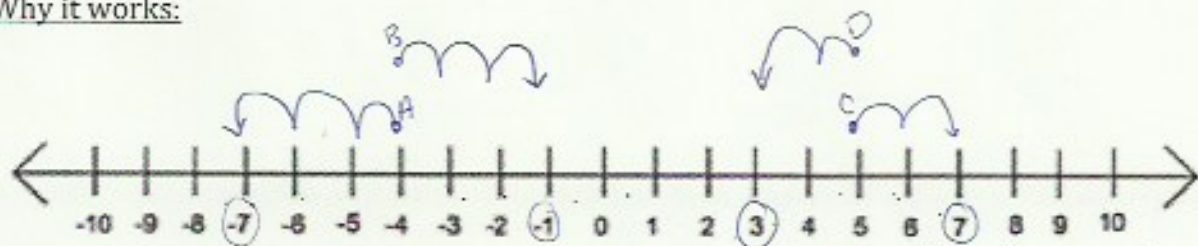
Different signs subtract

Keep the sign of the bigger number

Then you will be exact

$$\textcircled{B} \quad \begin{array}{r} -4+3 = -1 \\ 4-3 \end{array} \quad \begin{array}{r} +5-2 = +3 \\ +5-2 \end{array} \quad \textcircled{D}$$

Why it works:



Real life Application:

You owe four dollars and you spend three more now you owe -7 dollars

You **have** five dollars and you **get** two more now you have +7 dollars

You owe four dollars but you **pay** three now you owe -1 dollars

You **have** five dollars but you **buy** something for two now you have +3 dollars

White Board Practice

Multiplying and Dividing Negative and Positive integers

SAME sign= Positive answer $4 \times 4 = 16$ $-2 \times -2 = 4$

Different sign= Negative answer $4 \times -4 = -16$ $-2 \times 2 = -4$

Cheat Sheet-Term Rules 8nc9

Write products without the multiplication sign	$2 \times n$ $8 \times n = 8n$ $2n$
Write the number before the letter	$2n$ NOT $n2$ <i>except in addition subtraction</i>
Generally, write terms with letters before terms with numbers	$2n+4$ NOT $4+2n$
Generally, write terms in alphabetical order	$2c+2b \rightarrow 2b+2c$
When a term has more than one letter, write them in alphabetical order	$a \cdot c \cdot b \rightarrow acb \rightarrow abc$
Write negative terms after positive terms	$4-2a$ NOT $-2a+4$

* exponents take priority

$$x + x^2 \rightarrow x^2 + x$$

Simplify with like terms, Example:

$$2ab + ab - 5ab \rightarrow -2ab$$

$$3ab - 5ab \rightarrow -2ab$$

$$2y + 6y^2 - 3y^2 - 10y \rightarrow 2y - 10y - 8y + 6y^2 - 3y^2 \rightarrow -8y + 3y^2$$

Second set of cards (pyramid) pg.97

$3y^2 - 8y$ (exponents first, negatives in the back)

Quiz 8as3/8ea3

1. Copy and complete the table of values for the function machine

$$x \rightarrow +2 \rightarrow +10 \rightarrow y$$

X	2		10	
Y		13		25

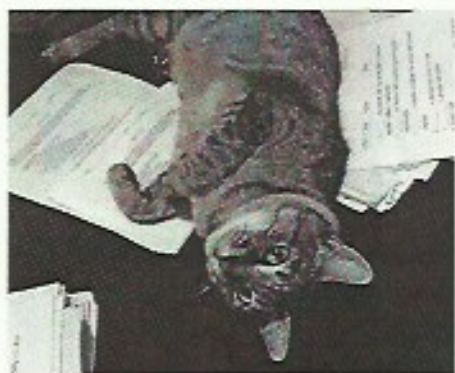
2. Write the formula for the function machine in problem 1
3. Work out the rule for the function machine

4	2
8	4
18	9

Denise thinks of a number, n

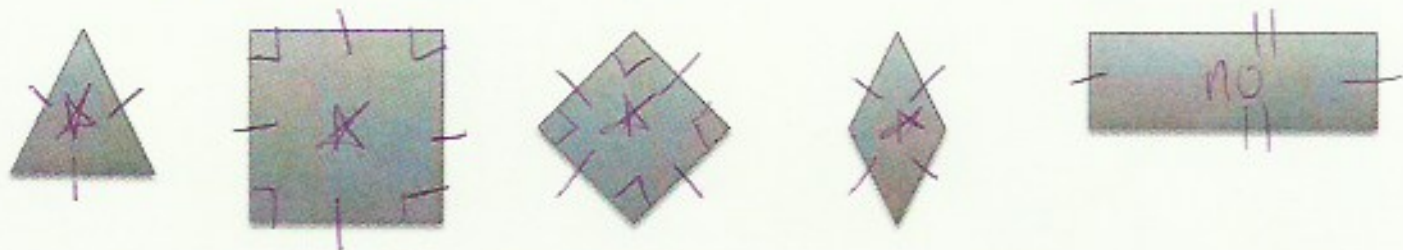
Write an expression for the number she gets when she:

4. Multiplies the number by 2 then adds 7
5. Divides the number by 3 then adds a 6



WRITING EQUATIONS

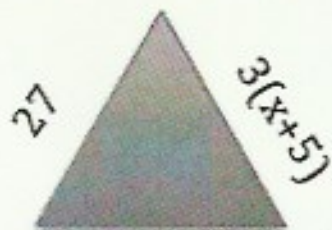
Review: Star the ones with equal sides



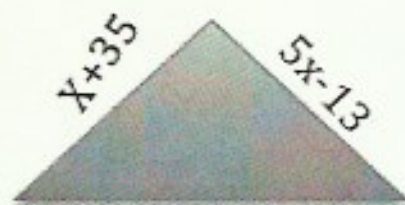
Construct: use information to write a formula

Example #1:

The diagram shows shapes. Work out the values of x and y .



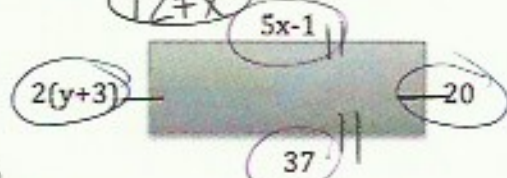
$$\begin{aligned}
 27 &= 3(x+5) \\
 -15+27 &= 3x+15-15 \\
 3 \div 12 &= 3x \div 3 \\
 \boxed{4} &= x
 \end{aligned}$$



$$\begin{aligned}
 x+35 &= 5x-13+13 \\
 \downarrow +13 & \\
 x+48 &= 5x-x \\
 -x & \\
 4 \div 48 &= 4x \div 4 \\
 \boxed{12} &= x
 \end{aligned}$$



$$\begin{aligned}
 4(y-3) &= 2y+2 \\
 4y-12 &= 2y+2 \\
 \downarrow +12 & \\
 4y &= 2y+14 \\
 -2y & \\
 \boxed{2y} &= \frac{14}{2} \\
 \boxed{y} &= 7
 \end{aligned}$$



$$\begin{aligned}
 2(y+3) &= 20 \\
 2y+6 &= 20 \\
 \downarrow -6 & \\
 2y &= 14 \\
 \boxed{y} &= 7
 \end{aligned}$$

$$\begin{aligned}
 5x-1 &= 37 \\
 \downarrow +1 & \\
 5x &= 38 \\
 \div 5 & \\
 \boxed{x} &= 7.6
 \end{aligned}$$

Check answers by substitution!